Achieving a Target Value for a Manufacturing Process: A Case Study

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A paint coating process for which there is little information known about the relationship between the production variables and the output variables was investigated. The goal was to determine the optimum process settings to achieve the target value for the response, coating thickness. Three planned experiments of eight runs each were carried out, the first with four replicates and the other two with two replicates. The observed data were analyzed using a multiple linear regression. Four input factors appeared to be significant and were used to set the production levels in such a way that the desired coating thickness was achieved.

Introduction

THE present paper reports a case study of how the response variable *paint coat thickness* depends on a set of six input factors. Since initially little was known about the relationship between factors and response, we decided to conduct three sequential experiments such that the second and third had improved experimental domains. The joint evaluation of all three experiments showed four factors to be significant. From the estimated response surface we were able to determine the factor settings necessary to achieve the desired target value of 0.8 mm paint coat thickness.

The Target: Paint Coat Thickness

The output variable of interest is the coating thickness resulting from a painting operation. Prior to the study the observed thickness varied between 2 mm and 2.5 mm, and exceeded the target value of 0.8 mm by a factor of two or three. The goal of the experiment was to determine the levels of the production factors that would produce the desired target value without substantially increasing the cost of production. Previous data on the process indicated that coating thickness varied considerably, but we failed to gather sufficient evidence to assign a numerical value to the standard deviation. Nor were we able to find out whether this variability could genuinely be attributed to the manufacturing process or whether it was due to the fact that each operator had their own way of adjusting the process towards the target value and that the work piece was produced in many different versions.

Hence, it was decided to build a prototype work piece that showed all the typical characteristics of the particular versions that the company would produce. It took a couple of months to design and produce the 140 requested prototype work pieces. Of these 140, our three experiments used a total of 64 pieces. In what follows, we report one observed response of paint coat thickness for each prototype work piece. Actually, each response itself is an average of 16 readings. All the prototype work pieces were analyzed by the same laboratory technician. The 16 readings per work piece were communicated to us about ten days after production. Because they showed little variation, it seemed reasonable to base the statistical analysis on the average readings reported later.

Potential Factors and Domains of Variation

Parallel to production of the prototype work pieces, the process engineers and machine operators were interviewed to obtain the input factors they thought might influence the response variable. As a first step, all factors that threatened to be costly or to lower the assembly line output were discarded. Additionally, other factors were eliminated if it was agreed that they would have little influence on the response, or if varying them on several levels would have been too costly. Six candidate factors emerged from the pre-experimental discussions. These factors, listed in de-

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creasing order of importance as later determined from an analysis of the data, are

A: belt speed
B: tube width
C: pump pressure
D: paint viscosity
E: tube height
F: heating temperature.

All factors could be varied continuously. Level 0 is the code for standard operating conditions. The factors were scaled so that levels ± 1 were expected to produce response changes that were detectable by the experiment. With this scaling it was technically feasible to vary factor levels in a range somewhere between -3 and 3 without increasing cost. The exact range depended on the individual factor.

Levels were easy to change for all factors except for paint viscosity. Full randomization over this factor was felt to be impossible as a practical matter. Therefore, we arranged the experiments so that the levels of paint viscosity were kept constant as long as possible. As a consequence, the replication variation presumably underestimates the true experimental error. However, since paint viscosity was varied only in the first and the second experiment, and the three observed standard deviations ($s_1 = 0.116$ mm, $s_2 = 0.153$ mm, and $s_3 = 0.104$ mm) were very nearly the same, there was no evidence that the lack of randomization of paint viscosity resulted in a gross estimation bias for the experimental error. We decided that we would make no special adjustment for the lack of randomization due to paint viscosity.

The First Experiment: Which Way to Go

The initial experiment was designed with each factor at one level above and one level below standard operating conditions. The purpose of such *screening experiments* was to obtain a gradient that indicated the direction to search for the target value (Box and Draper (1987, p. 12)). Since prior data indicated that the standard deviation might be large, we decided to run four replications for each of the eight runs of a 2_{III}^{6-3} fractional factorial design. This design is an orthogonal array of eight runs, for six factors of two levels each, of strength two. Observations 1–16 were taken on August 16, 1988 and observations 17-32 were taken on August 23, 1988. The design and data are given in Table 1.

Machine operators initially objected to randomization because they considered it too time consuming. Therefore, we implemented a kind of pseudo-randomization, which we could carry out ourselves without having to ask the operators. Namely, after each observation the levels of the factors (except for paint viscosity) were distorted and then readjusted. Since this required virtually the same effort as setting the factors at different levels, it helped us to persuade the engineers that randomization was *not* so time consuming. As a result everybody agreed to truly randomize the second and third experiments. For this first experiment we fitted a first-order model in the six factors *A-F*,

$$Y_{ij} = \theta_0 + \theta_1 A_i + \theta_2 B_i + \theta_3 C_i + \theta_4 D_i + \theta_5 E_i + \theta_6 F_i + e_{ij}$$

where $i = 1, \ldots, 8$ and $j = 1, \ldots, 4$. It turned out that tube height and heating temperature were not significant. The estimated response surface as a function of the remaining four factors (*A-D*) was

$$\hat{Y} = 1.42 - 0.32A + 0.21B + 0.21C + 0.07D. \quad (1)$$

An analysis of variance table for this and for the other two experiments is given in Table 4.

In order to study how the input factors affected process variability we also fitted a first-order model for $\log s_i^2$ (Box and Draper (1987, p. 285), Pukelsheim (1988), Taguchi (1988)). However, none of the factors were significant. Since most of the observed responses exceeded the target value of 0.8 mm, it was

TABLE 1. The First Experiment. The Design and the Observed Paint Coat Thickness [in mm]. Superscripts Indicate Time Order of the Observations

i	A	B	C	D	E	F	y_{i1}	y_{i2}	y_{i3}	y_{i4}
1	1	−1	1	−1	−1	−1	$^{1}1.09$	$^{2}1.12$	$^{3}0.83$	$^{4}0.88$
2	−1	−1	1	−1	1	1	$^{5}1.62$	$^{6}1.49$	$^{7}1.48$	$^{8}1.59$
3	1	1	−1	−1	−1	1	$^{9}0.88$	$^{10}1.29$	$^{11}1.04$	$^{12}1.31$
4	−1	1	−1	−1	1	−1	$^{13}1.83$	$^{14}1.65$	$^{15}1.71$	$^{16}1.76$
5	−1	−1	−1	1	−1	1	$^{17}1.46$	$^{18}1.51$	$^{19}1.59$	$^{20}1.40$
6	1	−1	−1	1	1	−1	$^{21}0.74$	$^{22}0.98$	$^{23}0.79$	$^{24}0.83$
7	−1	1	1	1	−1	−1	$^{25}2.05$	$^{26}2.17$	$^{27}2.36$	$^{28}2.12$
8	1	1	1	1	1	1	$^{29}1.51$	$^{30}1.46$	$^{31}1.42$	$^{32}1.40$

decided to run a second experiment with the experimental domain shifted appropriately.

The Second Experiment: An Incorrect Conjecture

Typically, the information about a response surface at a point outside the experimental domain is close to zero (Box and Draper (1987, p. 481)). Since the first experiment predicted that the target value of 0.8 mm was achieved outside the first experimental domain, we decided to run another experiment to check the predicted settings. In the first experiment the observed standard deviation of paint coat thickness was small enough that there was no need to run the second experiment with four replications. In the first experiment we had managed to realize 16 observations during one working day. Therefore, we decided that the second experiment with eight runs should have two replications. The second experiment was carried out on September 10, 1988.

The second design moved factors B, C, and D in a direction that would yield lower values for paint coat thickness. This is easily verified by the signs of the estimated coefficients in equation (1). For factor A, however, in spite of the fact that its negative coefficient (-0.32) indicated that its level should have been moved upwards, we actually moved it downwards. The reason for this was that we conjectured the following factor behavior.

The purpose of the belt was to accelerate the work piece to a high speed in order to shoot it through a continuous curtain of paint. From observing the process we believed that when the work piece was moved onto the belt the band slides under the work piece and does not transmit its full acceleration. Therefore, our conjecture was that *reducing* the belt speed would diminish the sliding effect and actually *increase* the acceleration of the work piece. However, the data from the second experiment showed otherwise (see Table 2). Again the 16 observations had paint coat thickness much above the target value, so we discarded our conjecture.

The second design was obtained from the first design using the fold-over method (Box and Draper (1987, p. 158)). Hence, the union of the two experiments would have provided an orthogonal design had we not shifted the levels of the second design so as to move in the direction of the target value. As a result of this shift the variances of the parameter estimates are no longer the same. There was, however, no evidence of a block effect between the first and the second experiment. Therefore, we proceeded with a joint evaluation of the union of the two data sets.

The response surface estimated from the total $32 + 16 = 48$ data points from the first and the second experiments was

$$\hat{Y} = 1.44 - 0.32A + 0.16B + 0.11C + 0.05D. \quad (2)$$

The striking similarity of equations (1) and (2) is an indication that the same model can be used to describe both the original domain of the first experiment and the shifted domain of the second experiment. The ANOVA table for the first two experiments combined is given in Table 4. Again we used the estimated response surface given in equation (2) to compute new target diagrams and to find settings for a third experiment at which the response was predicted to be close to the target of 0.8 mm.

The Third Experiment: Achieving the Target Value

Since it had proved efficient to make 16 observations per experiment, the third experiment was also planned to have eight runs with two replications. In view of what we had seen in the first two experiments, and since the process engineers were quite willing to fix paint viscosity at its low level, we decided to hold factor D fixed. This left us with the three factors A-C for which we ran a 2^3 complete factorial design. Since we had seen enough responses above the target, we chose the settings for the third experiment so that the responses would clearly fall below the target. The experiment was carried out on October 8, 1988 and is shown in Table 3.

Joint evaluation of all 64 data points gave the estimated response surface

$$\hat{Y} = 1.45 - 0.30A + 0.15B + 0.10C + 0.05D. \quad (3)$$

TABLE 2. Design and Data of the Second Experiment. The Top Eight Observations are Randomized, as are the Bottom Eight Observations. Superscripts Indicate Time Order of the Observations

i	A	B	C	D	y_{i1}	y_{i2}
1	−1.5	0	−2	0	$^{6}1.71$	$^{7}1.61$
2	0.5	0	−2	0	$^{3}0.91$	$^{8}1.30$
3	−1.5	−2	0	0	$^{1}1.71$	$^{2}1.60$
4	0.5	−2	0	0	$^{4}1.15$	$^{5}1.29$
5	0.5	0	0	−2	$^{9}1.33$	$^{16}1.06$
6	−1.5	0	0	−2	$^{10}1.74$	$^{13}1.98$
7	0.5	−2	−2	−2	$^{11}0.64$	$^{12}0.78$
8	−1.5	−2	−2	−2	$^{14}1.51$	$^{15}1.18$

TABLE 3. Design and Data for the Third, Randomized Experiment. Factor Levels are Chosen with Response Predicted to Fall Below the Target Value of 0.8 mm. Superscripts Indicate Time Order of the Observations

i	A	B	C	y_{i1}	y_{i2}
1	1.0	−2	−2	$^{6}0.57$	$^{15}0.58$
2	1.0	−1	−2	$^{9}0.62$	$^{16}0.74$
3	1.0	−2	−1	$^{1}0.75$	$^{3}0.58$
4	1.0	−1	−1	$^{7}0.79$	$^{11}1.04$
5	1.5	−2	−2	$^{12}0.51$	$^{14}0.66$
6	1.5	−1	−2	$^{2}0.69$	$^{5}0.49$
7	1.5	−2	−1	$^{4}0.53$	$^{8}0.64$
8	1.5	−1	−1	$^{10}0.78$	$^{13}0.79$

The corresponding analysis of variance table is shown in Table 4. Using equation (3) (with D fixed at -1) we computed a final target diagram of factor settings $A = 2 + \frac{1}{2}B + \frac{1}{3}C$ such that response was predicted to be on target. The overall design that incorporated all three experiments was nonorthgonal since the experimental domain was shifted from one experiment to the next and the spacings for the factor settings were unequal. As a consequence, the estimated coefficients in the fitted response surface equation did not have equal variance. Figure 1 shows, for four selected points in the experimental domain, the standard deviation of a predicted response of 0.8 mm. This suggested that factor level combinations 1, −2, 0, −1 in the bottom left corner or 2, 0, 0, −1 in the top right corner were preferable because of their small predictive standard deviation of 0.1371 mm and 0.1385 mm, respectively.

Conclusion

In retrospect we can count the number of factor level combinations that could have been used for our problem. Over all three experiments factor A was observed at five levels; factors B, C, and D were observed at four levels; and factors E and F were observed at two levels. Hence, the total number of possible factor level combinations was $5 \times 4^3 \times 2^2 = 1280$. Of these only 24 runs were realized, that is, two percent. This demonstrates the enormous savings that come from properly planned experiments. Also we used only 64 of the 140 prototype work pieces ordered, which was applauded by the process engineers as a proof of the economy of the approach chosen.

Since each factor was observed at more than two levels we were also able to investigate higher order terms and interactions. We found no improvement

TABLE 4. Analyses of Variance. Design and Data of the First Experiment with 32 Observations as Given in Table 1. The Second Analysis is for the 48 Observations From the First and the Second Experiment (Tables 1 and 2). The Third Analysis Includes the 64 Observations from all Experiments (Tables 1, 2, and 3)

	First Experiment	First and Second Experiment	First, Second and Third Experiment
Estimates [Coef/Std Dev]	[Coef/Std Dev]	[Coef/Std Dev]	[Coef/Std Dev]
Constant	1.42/0.021	1.44/0.022	1.45/0.021
A (belt speed)	−0.32/0.021	−0.32/0.020	−0.30/0.016
B (tube width)	0.21/0.021	0.16/0.019	0.15/0.016
C (pump pressure)	0.12/0.021	0.11/0.019	0.10/0.016
D (paint viscosity)	0.07/0.021	0.05/0.019	0.05/0.018
E (tube height)	−0.03/0.021		
F (heating temperature)	−0.01/0.021		
s/R^2	0.1161/0.94	0.1410/0.89	0.1328/0.93
Anova [SS/DF]	[SS/DF]	[SS/DF]	[SS/DF]
Regression	5.2475/6	6.9332/4	13.2720/4
Error	0.3371/25	0.8548/43	1.0407/59
Total	5.5846/31	7.7880/47	14.3127/63
Sequential SS [SS/DF]	[SS/DF]	[SS/DF]	[SS/DF]
A (belt speed)	3.264/1	4.0964/1	9.4808/1
B (tube width)	1.345/1	1.9495/1	2.8129/1
C (pump pressure)	0.456/1	0.7480/1	0.8538/1
D (paint viscosity)	0.154/1	0.1393/1	0.1244/1
E (tube height)	0.022/1		
F (heating temperature)	0.007/1		

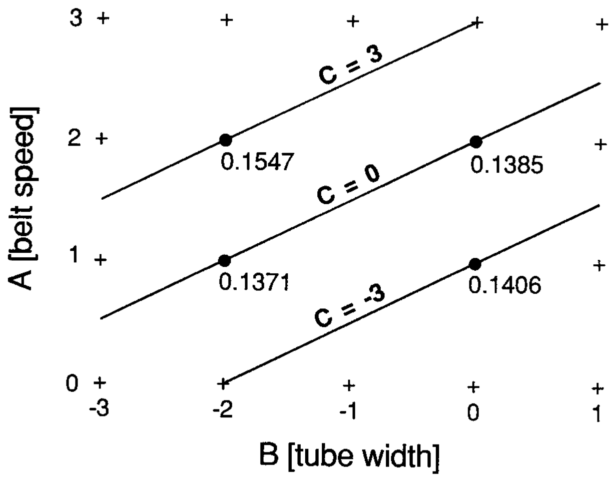

FIGURE 1. Target Diagram. Level of Factor A [Belt Speed] as a Function of B [Tube Width] and C [Pump Pressure], With D [Paint Viscosity] at Level −1, so as to Achieve the Target Value of 0.8 mm Paint Coat Thickness. Predictive Standard Variation is not Constant, and is Given at Four Selected Level Combinations.

over the fitted equation (3), despite the fact that there is a significant *BC* interaction in the third experiment alone. Since the final target diagram shown in Figure 1 proved to work well, it was concluded that from a practical point of view, interactions were negligible.

We rejected the idea of running a final confirmation experiment for a number of reasons. First, we wanted to finish the experimental series before Christmas. Second, the data given here are coatings of the top side of the work piece. In the actual experiment we also observed and evaluated coatings of the bottom side. For the bottom side the target value was achieved at the end of the second experiment, and the third experiment then merely served as a confirmation experiment. The results achieved for the bottom side were promising enough that we did not feel it was necessary to run another confirmation experiment for the top side.

The experiments resulted in operating conditions that achieved the target value of 0.8 mm with an average standard deviation of 0.1 mm. This compared favorably with the conditions prior to the study. The increased knowledge of the process proved to be helpful when, shortly afterwards, a new paint was introduced. Without formal experimentation the engineers were able to find factor settings that brought the process on target.

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